Magnesium
Editor-in-Chief: B.M. Altura, Brooklyn, N.Y.

Reprint
Publisher: S. Karger AG, Basel
Printed in Switzerland

Magnesium *2:* 93–104 (1983)

0252–1156/83/0022–0093$2.75/0

Predictors of Maximum Voluntary Contraction Force of Quadriceps femoris Muscle in Man[1]

Ridge Regression Analysis

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Key Words. Serum magnesium concentration · Maximum voluntary contraction force · Ridge regression analysis

Abstract. 106 subjects (mean age 43.9, SD 13.3, range 19–83 years; 88 males and 18 females), comprising 93 consecutively admitted patients (45 chronic alcoholics, 27 chronic alcoholics with concomitant disease and 21 nonalcoholics with somatic disease) and 13 normal controls, were interviewed to assess their level of physical training, tested for their maximum voluntary contraction force (MVC) of the quadriceps femoris muscle and screened for 19 laboratory constituents. Anthropometric measurements consisted of weight and height in 64 cases and of measurement of the length of tibia in 63 cases.

Mean MVC of the 93 patients ($\bar{x}$ 39.3, SD 15.6, range 1–94 kp) differed significantly from that of the controls ($\bar{x}$ 56.2, SD 8.9, range 42–68 kp); $p < 0.001$. The following laboratory variables were significantly correlated to MVC: serum magnesium (S-Mg); $p < 0.001$, serum sodium, serum iron, serum ALAT; $p < 0.01$ and blood standard bicarbonate; $p < 0.05$. Mean S-Mg of the 93 patients ($\bar{x}$ 0.716, SD 0.101, range 0.43–0.91 mmol/l) differed significantly from that of the controls ($\bar{x}$ 0.808, SD 0.061, range 0.71–0.93 mmol/l); $p < 0.001$.

In order to assess more fully the relationship of S-Mg and MVC, taking into account other factors known to affect MVC (e.g., anthropometric data, the level of training and diagnoses), 36 cases, with full information available for the 10 potential explaining variables, were analyzed using a variant of multiple regression analysis: ridge regression. The three statistically significant predictors of MVC were: weight (β-weight +0.510; $p < 0.001$); age (β-weight −0.433; $p < 0.001$) and S-Mg (β-weight +0.309; $p < 0.01$). In conclusion: following weight and age, S-Mg is the only additional, statistically significant predictor of MVC.

[1] This study was supported by Karolinska Institute research grant No. LM-210-5 and by a research fellowship to one of the authors (G.S.L.).

Introduction

Earlier studies suggested a relationship between serum magnesium concentration (S-Mg) and the maximum voluntary muscle contraction force (MVC) of the quadriceps femoris muscle in man [16, 17]. In order to validate earlier results, the present study was designed to include more patients and more diagnostic entities, other laboratory constituents in addition to S-Mg and factors which are known to affect MVC, e.g., anthropometric data and the individual level of physical training. To provide a broad range of S-Mg values, the study included patients with diagnoses of diseases in which hypomagnesemia had been reported [20], and apparently healthy subjects.

The purpose of the present work was (1) to confirm the relation between S-Mg and MVC suggested by earlier studies and (2) to assess the relative importance of S-Mg for MVC, while considering, at the same time, the contribution of other potential explaining variables. In order to achieve this, a conventional statistical screening was followed by the use of a new statistical technique, ridge regression (see Appendix).

Material and Methods

Material I: Source Population (n = 106)

The source population (table I) consisted of 106 subjects comprising three major diagnostic groups of patients (table II) and a group of apparently healthy subjects (table III), 88 males and 18 females (mean age 43.9, SD 12.6, range 19–83 years). The patients were selected from (a) alcoholics consecutively admitted to the Alcohol Department of the Karolinska Hospital and outpatients of the Maria Clinic, Stockholm (groups I and II; table I), and (b) patients admitted to various departments of the Karolinska Hospital with diagnoses of diseases in which hypomagnesemia was reported in the literature, predominantly from endocrine and internal medicine departments (groups II and III; table I). The apparently healthy male subjects were selected randomly to serve as controls (group IV; table I).

Material II: Population Analyzed by Ridge Regression (n = 36)

23 male patients and 13 male controls (table IV) were selected by the computer program for analysis by ridge regression technique (see Statistics).

S-Mg Estimation

S-Mg was estimated by atomic absorption spectrophotometry at the Clinical Chemistry Laboratory of the Karolinska Hospital, using a Perkin-Elmer spectrometer No. 403 (coefficient of variation 1.18%). Simultaneous screening of the patients for 18 additional laboratory constituents was carried out at the same laboratory (table V).

MVC Measurements

The measurement of MVC was carried out as soon as possible following admission to the hospital to eliminate a possible lowering of MVC subsequent to prolonged bed rest. Patients who were bedridden for a period of more than 2 weeks prior to admission were excluded, except in the rare case in which the muscle weakness was the presenting symptom and was so extreme as to necessitate the recumbency. If delay was imposed for medical reasons, only such cases were included as had a regular daily training program in the recumbent position, including training of the quadriceps muscle.

Table I. Source population (n = 106)

No.	Group	n
I	Alcoholics with no concomitant disease	45
II	Alcoholics with concomitant disease	27
III	Nonalcoholics with one or more diagnoses	21
IV	Apparently healthy controls	13
	Total	106

Table II. Nonlaboratory variables; all patients (n = 93)

Variable	Variable No.	$\bar{x}$	SD	SEM	n	Range
Age, years	1	45.6	12.6	1.3	93	19–83
Weight, kg	2	76.6	12.1	1.7	51	45–104
Height, cm	3	176.0	6.8	0.9	51	161–190
Length of tibia, cm	9	40.3	2.7	0.4	50	31–47
MVC, kp (= 9.81 N)	11	39.3	15.6	1.6	93	1–94

Dummy variables: level of training (variables 4, 5): high, 40 patients; moderate, 23 patients; untrained, 30 patients. Diagnoses (variables 6–8): alcoholics without concomitant disease: 45; alcoholics with concomitant disease: 27; nonalcoholics with one or more diagnoses: 21 patients. Sex (variable 10): 75 males and 18 females.

Table III. Apparently healthy control subjects (males; n = 13)

Variable	Variable No.	$\bar{x}$	SD	SEM	Range
Age, years	1	31.2	11.3	3.1	20–56
Weight, kg	2	65.0	10.3	2.9	52–87
Height, cm	3	175.5	7.6	2.1	165–192
Length of tibia, cm	9	38.6	3.3	0.9	31–43
S-Mg, mmol/l	12	0.808	0.061	0.017	0.71–0.93
MVC, kp	11	56.2	8.9	2.5	42–68

Dummy variables: level of training (variables 4, 5): high, 8 subjects; moderate, 3 subjects; untrained, 2 subjects.

Table IV. Variables used for the ridge analysis (males; n = 36)

Variable	Variable No.	$\bar{x}$	SD	SEM	Range
Age, years	1	40.0	14.1	2.3	19–73
Weight, kg	2	72.0	11.2	1.9	52–95
Height, cm	3	176.0	7.1	1.2	165–192
Length of tibia, cm	9	40.0	2.8	0.5	31–47
MVC, kp	11	52.2	2.1	12.9	25–94
S-Mg, mmol/l	12	0.751	0.091	0.015	0.57–0.93

Dummy variables: level of training (variables 4, 5): high, 17 patients; moderate, 12 patients; untrained, 7 patients. Diagnoses (variables 6–8): alcoholics without concomitant disease, 11; alcoholics with concomitant disease, 9; nonalcoholics with one or more diagnoses, 3; controls, 13.

Table V. Laboratory variables; all patients (n = 93)

Variable	Variable No.	$\bar{x}$	SD	SEM	n	Range of patients' values	Laboratory's range of normal values
S-magnesium, mmol/l	12	0.716	0.101	0.011	93	0.43–0.91	0.72–0.90
S-potassium, mmol/l	13	4.17	0.63	0.08	63	2.9–6.1	3.5–4.9
S-sodium, mmol/l	14	140.0	3.1	0.4	62	132–154	137–145
S-chloride, mmol/l	15	101.0	4.4	1.3	11	92–107	95–107
B-standard bicarbonate, mmol/l	16	27.0	3.7	0.5	60	20–36	23–32
S-phosphate, mmol/l	17	1.15	0.22	0.03	55	0.5–1.6	0.8–1.5
S-calcium, mmol/l	18	2.365	0.119	0.021	33	2.12–2.66	2.20–2.60
S-iron, µmol/l	19	17.37	10.57	1.97	29	4.7–42.6	14.0–32.0
S-creatinine, µmol/l	20	90.6	46.7	6.7	48	47–371	53–124
S-albumin, g/l	21	40.0	4.4	0.6	53	33–60	39–52
B-sugar, mmol/l	22	5.02	0.96	0.18	28	3.6–7.6	3.3–5.5
S-cholesterol, mmol/l	23	6.31	2.16	0.49	24	3.9–14.4	3.9–16.5
S-triglycerides, mmol/l	24	1.46	0.71	0.17	18	0.7–2.7	0.9–1.5
S-bilirubin, µmol/l	25	13.00	8.68	1.54	32	3.4–44.5	3.4–13.7
S-ASAT, U/l	26	31.5	40.3	5.8	49	7–225	<40
S-ALAT, U/l	27	24.0	21.4	3.1	49	4–104	<35
S-LDH, U/l	28	245.7	70.2	15.3	21	132–424	<200
S-ALP, IU/l	29	37.4	17.2	2.7	41	17–95	<85

The MVC was measured using a modification of the method of *Tornvall* [19] developed at the Military Medical Examination Center, Stockholm. The chair used was equipped with a strain gauge dynamometer, connected by a band placed at the level of the lateral malleolus. The subject's pelvis and chest were held by a belt to prevent simultaneous movement of the hip joint. The MVC of the quadriceps muscle was measured separately in each leg. The patient was instructed to attempt maximal knee extension against the constraining band and encouraged by repetitive intensive suggestions to sustain a maximum level of effort throughout. The maximal contraction, sustained for at least 2–3 s, was recorded. The measurements were repeated three times consecutively, on each side, with an interval of 1–3 min. If the highest value was obtained on the last of the three trials, an additional contraction was requested. The highest value obtained on at least three trials represented the MVC, expressed in kiloponds. The results were either recorded on a UV recorder or read directly from the dial of recording instrument (a modified Bofors transducer indicator type BK-1).

When evaluating the MVC from the UV strip chart, the highest plateau obtained during at least 2 s of the most forceful contraction was taken as valid. The overshoot spikes were disregarded. When the results were read off directly from the dial, the highest value shown by the pointer was recorded.

Anthropometric Measurements

The length of the tibia was measured with a soft measuring tape between the margin bordering on the tibiofemoral articular groove and the distal end of the medial malleolus. Weight and height were measured in a standard manner.

Assessment of the Level of Training

The individual level of physical training was assessed by means of a standardized interview which aimed at obtaining full information from every subject concerning working capacity and the type, duration and frequency of physical activity during leisure.

Subjects who were not fit enough to work or carry out any physical activity were classified as untrained. Subjects who were capable of working or of walking 2 km daily or carrying out an outdoor sport activity at least three times weekly were classified as moderately trained. Finally, subjects who were able to work and in addition walk 2 km or more daily, or carry out an outdoor sport activity, were considered highly trained. The source population, according to these criteria, consisted of 30% untrained, 41% moderately trained and 20% highly trained subjects.

The MVC and the tibial measurements, as well as the assessment of the level of training, were made by the same observer (G.S.L.) to secure standardized test conditions.

Statistical Analysis

In view of the need to screen a very large number of potential explaining variables (namely 60), means, standard deviations and Pearson correlation coefficients were calculated from the source population, from each of the four groups separately, and from the combined patient group. Since the data were not complete for all subjects (18 laboratory constituents, additional to S-Mg, were available in 11–63 patients, the length of tibia in 63 and the height and weight in 64 out of 106 subjects), the correlation coefficient could not be calculated for each pair of variables *for all subjects*. As a substitute, to be used only in the screening process, the correlation coefficients were calculated using every possible pair of observations, so that the results were based upon different numbers of subjects. Since, for technical reasons, all 60 variables could not be used in the ridge regression, a preliminary choice of potential explaining variables was made by detailed examination of the Pearson correlation matrix, and on the basis of substantive knowledge.

Six laboratory constituents, in addition to S-Mg, appeared in this short list. Since ridge regression, like any other multiple regression technique, should be used with complete observation vectors (all correlation coefficients based on the same subjects), submission of the short list to a ridge regression with complete observation vectors reduced the number of subjects to such an extent that the results were not reliable. In order to increase the number of subjects available, the laboratory constituents other than S-Mg had to be dropped from the short list which was now reduced to 11 potential explaining variables.

The qualitative variables on the short list, namely, diagnostic groups, level of training, and sex, were expressed as dummy variables [7, 18].

Among the 41 cases with complete observations for the MVC and 11 potential explaining variables, there were only 5 female subjects. As there were too few females, the factor of sex (former variable 60; variable 10 in tables II and VI) was dropped from the short list. The ridge regression was consequently applied to 36 male subjects, made up of 23 patients and 13 apparently healthy controls (table IV), referred to as material II, using 10 potential explaining variables.

The higher MVC value of the two sides was used as the dependent variable (variable 63 in the original screening, variable 11 in tables II–IV, VI and VII), as it is more consistent with the definition of MVC than the mean MVC of both sides (variable 64 in the original screening). Since the correlation between these two alternative MVC values was $r = 0.996$, no practical difference in the results would follow.

Results

Material I

The Nonlaboratory Variables. The age, sex, anthropometric measurements, level of training, diagnostic entities, and the MVC of the patient population are seen in table II, and of the apparently healthy subjects, in table III.

The Laboratory Variables. The laboratory results of the patients' screening are seen in table V and the S-Mg of the controls in table III.

MVC Compared. The MVC of all the patients ($\bar{x}$ 39.3, SD 15.6, n = 93, range 1–94 kp) and that of the controls ($\bar{x}$ 56.2, SD 8.9, n = 13, range 42–68 kp) differed significantly ($p < 0.001$, Student's t test). When each diagnostic group was compared separately to the controls, the MVC of each group (I: $\bar{x}$ 44.5, SD 12.8, n = 45, range 22–94 kp; II: $\bar{x}$ 37.6, SD 16.8, n = 27, range 6–64 kp; III: $\bar{x}$ 30.2, SD 16.0, n = 21, range 1–74 kp) differed significantly from that of the controls

Table VI. Significant correlations of MVC (variable 11); all patients (n = 93)

Variable	Variable No.	Correlation	Number of patients	p
Age	1	−0.3662	93	<0.001
Weight	2	+0.3256	51	<0.05
Height	3	+0.4931	50	<0.001
Length of tibia	9	+0.3548	50	<0.01
Sex	10	+0.4841	93	<0.001
High training	4	+0.2545	93	<0.01
Moderate training	5	+0.3414	93	<0.001
Alcoholics without concomitant disease	6	+0.3278	93	<0.001
Nonalcoholics with one or more diagnoses	8	−0.3163	93	<0.001
S-magnesium	12	+0.3270	93	<0.001
S-sodium	14	+0.3629	62	<0.01
B-standard bicarbonate	16	+0.2676	60	<0.05
S-iron	19	+0.4957	29	<0.01
S-ALAT	27	+0.3544	49	<0.01

Table VII. Results of ridge analysis (males; n = 36)

Variable	Variable No.	β-weights	Standard error	p
Weight	2	+0.5096	0.1099	<0.001
Age	1	−0.4334	0.1058	<0.001
S-magnesium	12	+0.3090	0.1011	<0.01
Length of tibia	9	−0.1661	0.1111	n.s.
High training	4	+0.1204	0.1078	n.s.
Alcoholics with concomitant disease	7	−0.0732	0.1079	n.s.
Alcoholics without concomitant disease	6	−0.0425	0.1043	n.s.
Moderate training	5	+0.0370	0.1118	n.s.
Nonalcoholics with one or more diseases	8	−0.0317	0.1042	n.s.
Height	3	+0.0254	0.1136	n.s.

Dependent variable: MVC (variable 11), total explanation; 68.4%, k = 0.15.

($p < 0.001$, Student's t test). Within the three diagnostic groups, only groups I and II differed significantly ($p < 0.001$, Student's t test).

S-Mg Compared. The mean S-Mg of the patients and of the controls (tables III, V) differed significantly ($p < 0.01$, Student's t test). The mean S-Mg of each diagnostic group (I: $\bar{x}$ 0.728, SD 0.103, range 0.45–0.91; II: $\bar{x}$ 0.707, SD 0.112, range 0.43–0.91; III: $\bar{x}$ 0.705, SD 0.085, range 0.54–0.86) differed significantly from that of the controls ($p < 0.05$, $p < 0.001$ and $p < 0.001$, respectively; Student's t test).

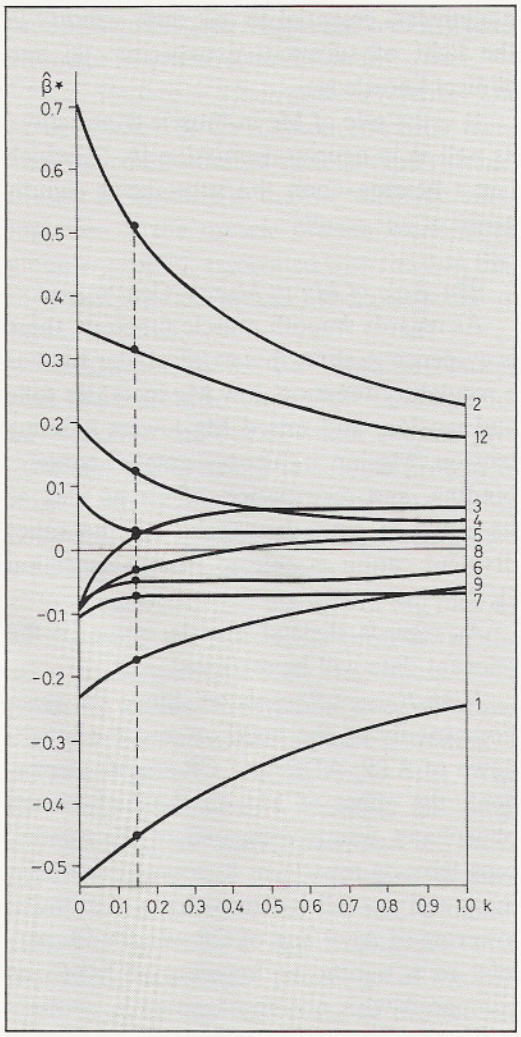

Fig. 1. Ridge regression coefficients (β-weights).

There was no statistically significant difference in mean S-Mg on comparison within the three diagnostic groups.

Relation of MVC to Other Variables. The statistically significant correlation coefficients of MVC of all the patients are seen in table VI.

Relation of S-Mg to Other Variables. In addition to its positive correlation to MVC (table VI), S-Mg was significantly negatively correlated with S-calcium ($r = -0.3266$, $n = 33$, $p < 0.05$), positively with S-potassium ($r = +0.2092$, $n = 63$, $p < 0.05$) and S-albumin ($r = +0.2617$, $n = 53$, $p < 0.05$) and negatively with S-ASAT ($r = -0.3179$, $n = 49$, $p < 0.01$).

Material II

Ridge Regression Results. The results are presented in table VII and figure 1. The only statistically significant independent variables found by the ridge regression, and the most important as measured by the absolute magnitude of the standardized regression coefficient (β-weight, see Appendix) were weight (variable 2), $p < 0.001$, age (variable 1), $p < 0.001$ and S-Mg (variable 12), $p < 0.01$.

The standardized regression coefficient for weight has a positive sign (+0.5096), which means that the heavier the subject the greater the MVC. For age the sign is negative (−0.4334), so that the older the subject, the lower the MVC. The third significant factor, S-Mg, has a positive sign (+0.3090), showing that the higher the serum magnesium concentration, within the range measured (table IV), the higher the MVC. In addition to the 3 factors found to be statistically significant, the model included 7 other factors (table VII). The 10 variables together gave a total explanation of 68.4%.

Clinical Observations during the Test Situation

The majority of the patients showed marked fatigue during the test situation. Some began to perspire and some complained of vertigo. Coarse finger tremor, present in a number of patients, became aggravated, and a few patients developed a

marked tremor of the lower limbs, necessitating a pause of up to 3 min between repeat measurements (see Methods).

Discussion

The two most important predictors of MVC in our ridge analysis proved to be age and weight. Age has been known to affect MVC [3] and so has weight [19]. Our results, however, show the hierarchic importance of these two factors among the anthropometric and other factors considered.

The fact that body weight was shown to be far more important than body height and the length of tibia is in agreement with the finding of *Tornvall* [19] of a highly significant correlation between weight and the isometric muscle strength and of a nonsignificant correlation between the latter and body height as well as tibia length. However, the fact that the level of physical training had no significant effect on MVC was an unexpected finding. Furthermore, the ridge analysis results show S-Mg to be the only additional predictor of MVC, third in the order of importance, even when considered simultaneously with other potentially explaining factors. The ridge analysis results suggest, therefore, a major role for S-Mg (i.e. extracellular magnesium) in determining the MVC of skeletal muscle in man.

S-Mg is positively related to MVC, i.e., the higher the S-Mg within the range measured, the higher the MVC. The conclusions are limited to men only, since females were excluded in the final run, but the MVC of females may be extrapolated from that of the males [3].

No matter how clear-cut the results of a statistical analysis may prove to be, it is nonetheless essential to test their validity in the light of substantive experimental and clinical knowledge.

It is the role of Mg in muscle contraction, as well as in neurotransmission [4, 5], which has a bearing upon the substantive significance of our results.

The Role of Mg in Muscle Contraction

As regards smooth muscle function, there is evidence to show that extracellular Mg has a regulatory function; low Mg increases tone and tension, and raised Mg lowers baseline tension. Mg ion influences uptake, content, binding, and distribution of Ca as well as its efflux in smooth muscle, and no other divalent cation is able to mimic its action [1, 2].

As regards skeletal muscle, some of the relevant data will be reviewed.

High-Energy Phosphates. Since the reactions involving the build-up and the breakdown of ATP, ADP, and CP are Mg-dependent, the effect of Mg on the high-energy phosphates was investigated in the quadriceps femoris muscle in hypomagnesemia in man, and a small but statistically significant lowering of ADP and of CP was found, parallel to a significant lowering of MVC, as compared with normomagnesemic controls [17].

Sarcoplasmic Reticulum Ca Transport. The sarcoplasmic reticulum Ca transport and the simultaneous ATP hydrolysis, which provides its energy requirement, depend on the action of the Ca^{2+}-ATPase (the calcium pump) of the membranes of the sarcoplasmic reticulum. The activity of the Ca^{2+}-ATPase requires the presence of Mg ions. The Ca^{2+}-ATPase interacts with ATP chelated with Mg. The synthesis of ATP from ADP during Ca efflux (the work in reverse of the calcium

pump) requires also the presence of Mg ions, although in low concentration [8].

Ca Binding – Contractile Proteins. The myoplasmic free Ca ion concentration regulates the contraction of skeletal (and cardiac) muscle by binding to the Ca^{2+}-specific binding sites of the muscle protein troponin (a globular protein, associated with the thin filaments of striated muscle, and composed of three subunits).

In addition to the Ca^{2+}-specific binding sites there are also Ca^{2+}-Mg^{2+} binding sites on troponin, myosin (a major muscle protein) and parvalbumin (a soluble protein of muscle sarcoplasm), which bind Ca and Mg competitively. These latter sites are saturated with Mg in a relaxed muscle.

It is suggested that the role of Mg may be to maintain the contractile proteins in the same conformational state, regardless of Ca fluxes. In the case of troponin, this conformation may be a precondition for Ca binding to the Ca^{2+}-specific binding sites [13]. The Ca^{2+}-Mg^{2+} binding sites maintain the integrity of the troponin molecule, and the binding of free Ca and Mg ions at these sites affects the interaction of the two troponin subunits; troponin C and troponin I. This interaction is thought to be one of the primary steps in the regulation of the muscle contraction cycle [21].

An important event in muscle contraction is the interaction of the two major muscle proteins myosin and actin to form an actinomyosin complex, which exhibits a high ATPase activity in the presence of free Mg ions. The substrate for actinomyosin ATPase is MgATP. The resulting ATP hydrolysis supplies the energy for muscle contraction.

The experimental work of *Kardami and Gratzer* [11] suggests that the low molecular weight myosin subunits, i.e., the myosin light chains, which are thought to play a part in binding and release of free Ca ions, undergo a conformational change as a result of binding of free Ca, or Mg ions at its Ca^{2+}-Mg^{2+} binding sites, thereby possibly facilitating the interaction of myosin with actin.

In yet another contractile protein, calsequestrin, contained in the terminal cisternae of the sarcoplasmic reticulum [15], it has been shown by means of electron probe analysis, using frog skeletal muscle rendered tetanic, that the Ca release was accompanied by a small but significant influx of Mg and potassium. Since calsequestrin also binds Mg, the authors suggest that during muscle activation, Mg enters the sarcoplasmic reticulum membrane (either as a counter ion, or by active transport).

Mg in Neurotransmission

Experimental evidence has shown that at the myoneural junction, Mg antagonizes the action of Ca by blocking its influx into the nerve terminal following an action potential and at rest; and that Mg initially activates and at increasing concentrations depresses subsynaptic cholinergic receptors [4].

Singh et al. [14] reported a presynaptic effect of Mg; a decrease of the quantal content of the excitatory endplate potential, as well as a postsynaptic effect; a decrease in the frequency as well as in the amplitude of the miniature endplate potential.

Apart from the need of Mg and ATP in the synthesis of neurotransmitters, it is the role of Mg at the myoneural junction – which indicates that hypomagnesemia should increase transmitter release and neuromuscular hyperexcitability [5] – that is of major relevance in support of the substantive validity of our statistical results.

The increased neuromuscular excitability was also demonstrated by the clinical signs shown by several patients during the test situation.

Appendix

The technique of ridge regression, an adaption of multiple regression, is fully explained by *Hoerl and Kennard* [9, 10], has been applied by *McDonald and Schwing* [12] to the problem of mortality from air pollution, and has been compared with other regression techniques by *Dempster* et al. [6]. The need to modify multiple regression stems from its deficiencies in the presence of correlation (nonorthogonality) among the independent variables themselves, which may produce misleading results.

The lack of orthogonality in multiple regression has four deleterious consequences:

(a) Estimates of the regression coefficients tend to be too large;

(b) Despite the tendency to overly large regression estimates, their standard errors may be highly inflated, making difficult the detection of statistically significant coefficients;

(c) Signs of the regression estimates (+ or –) may be opposite to those suggested by substantive knowledge;

(d) Estimated regression coefficients may be unstable in the sense that slight changes in the observations may produce large changes in the regression estimates.

Despite the presence of correlation, ordinary multiple regression estimates retain the properties of being unbiased and of producing a minimum standard deviation of the residuals from the regression prediction. It can be said that ordinary multiple regression achieves these desirable results at the expense of an inflated standard error for the estimated regression coefficients.

Ridge regression overcomes the four listed shortcomings of multiple regression by deliberately introducing a small amount of bias and thereby reducing considerably the inflated standard errors of multiple regression. The net result is that the root-mean-square error of the estimates (the vector sum of bias and standard error) is made smaller than that for ordinary multiple regression, which is composed entirely of an inflated standard error. At the same time, although the residual sum of squares from the regression (the 'unexplained' part) is slightly increased, the regression estimates as a group are made smaller in absolute value (shrunk), illogical signs of regression coefficients are reversed, standard errors are reduced, and the coefficients themselves are more stable (less sensitive to minor changes in the data).

The mechanics of ridge regression are straightforward. Instead of using the ordinary matrix of Pearson correlation coefficients for the independent variables, which is central to all multiple regression techniques, the diagonal of 1s of that matrix is modified to 1+k, where k represents given values between 0 and 1, in small steps of, say, 0.05. The multiple regression model is solved repeatedly for the different values of k, so that a number of sets of regression estimates are obtained, one set for each value of k used. In matrix notation, the solutions for the ridge regression coefficients for a given k, $\hat{\beta}^*$ (k), are given by

$$\hat{\beta}^*(k) = (R_{xx}+kI)^{-1} R_{xy},$$

where R_{xx} is the ordinary correlation coefficient matrix for the independent variables, kI is a diagonal matrix with k on the diagonal, and R_{xy} is the vector of correlation coefficients of the dependent variable with each of the independent variables.

The effect of working directly with the correlation matrix is to give estimated ridge regression coefficients in standardized form, $\hat{\beta}_i^*(k)$, the so-called beta weights. Beta weights can be converted to ordinary regression coefficients by multiplying each one by the ratio of the standard deviation of the dependent variable to the standard deviation of the appropriate independent variable. Interpretation of the ridge regression coefficients (beta weights) is direct: the larger the absolute value of the coefficient the more important the independent variable as a predictor of the dependent variable. Statistical significance of the regression coefficient is tested for in the usual way, by comparing the estimate with its standard error.

The complex interrelationship among the independent variables due to nonorthogonality is exhibited in a graph of the $\hat{\beta}_i^*(k)$, the standardized regression coefficients as functions of k, called the ridge trace (fig. 1). This graph is used to select the value of k at which the system of regression coefficients becomes reasonably stable.

As k is varied from zero (the ordinary multiple regression case), the magnitudes of the regression estimates as a group are reduced, some to insignificant values, and some originally 'incorrect' or illogical signs

may even be reversed. The system as a whole tends to stabilize for some value of k, usually in the range 0.1–0.3.

For the problem at hand, the system tends to stabilize in the neighborhood of k = 0.15, which is shown in figure 1 as a vertical dotted line. The intersections of the vertical line with the curves for the standardized regression coefficients, $\hat{\beta}_i^*(k)$, give the regression results which are also shown in table VII. At the value of k = 0.15, the regression estimates should be closer to the true regression coefficients and more suitable for estimating individual effects than the estimated coefficients given by ordinary multiple regression (k = 0).

Facteurs permettant de prévoir la force de contraction volontaire maximale (CVM) du quadriceps fémoral chez l'homme. Analyse des pentes de régression

106 cas (âge moyen = 43,9 ans, DS = 13,3, extrêmes = 19–83 ans; 88 hommes et 18 femmes) comprenant 93 malades hospitalisés (45 alcooliques chroniques, 27 alcooliques chroniques avec une autre affection associée, 21 non alcooliques avec des maladies somatiques) et 13 témoins normaux ont été interrogés pour établir leur taux d'entraînement physique, testés pour mesurer la force de contraction volontaire maximale (CVM) du quadriceps fémoral et dosés pour 19 données de laboratoire. Les mesures anthropométriques ont porté sur le poids et la taille dans 64 cas et sur la mesure de la longueur du tibia dans 63 cas. La CVM moyenne des 93 malades ($\bar{x}$ = 39,3; DS = 15,6; extrêmes = 1–94 kp) diffère significativement ($p < 0,001$) de celle des témoins ($\bar{x}$ = 56,2; DS = 8,9; extrêmes = 42–68 kp). Les données de laboratoire suivantes sont corrélées à la CVM: Mg sérique (Mg s) ($p < 0,001$); Na s, sidérémie, ALAT s ($p < 0,01$); et bicarbonate sanguin standard ($p < 0,05$). La moyenne du Mg s des 93 malades ($\bar{x}$ = 0,716, DS = 0,101, extrêmes = 0,43–0,91 mmol/l) diffère signifiicativement ($p < 0,001$) de celle des contrôles ($\bar{x}$ = 0,808, DS = 0,061, extrêmes = 0,71–0,93 mmol/l). Dans le but d'établir plus complètement l'interrelation entre Mg s et CVM en prenant en considération d'autres facteurs connus pour affecter la CVM (par ex. des données anthropométriques, le niveau d'entraînement, les diagnostics), 36 cas, avec les informations concernant 10 variables potentielles, ont été analysés en utilisant un type d'analyse de régression multiple: la régression des pentes. 3 facteurs permettent de prévoir la CVM: le poids (β poids: +0,510, $p < 0,001$), l'âge (β poids: −0,433, $p < 0,001$) et le Mg s (β poids: +0,309, $p < 0,01$). En conclusion: après le poids et l'âge, le Mg s est le seul facteur statistiquement significatif permettant de prévoir la CVM.

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Received: April 26, 1983
Accepted: July 11, 1983

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